Discovery Exercise for Tensors

Figure 1 shows a vector $\vec{V}$ and two sets of axes. The exact components of the vector are not important for this problem, but it is important that the $x'y'$ axes are at a 45° angle relative to the xy axes. Let V_x and V_y be the components of $\vec{V}$ in the xy basis, and let $V_{x'}$ and $V_{y'}$ be the components of $\vec{V}$ in the $x'y'$ basis.

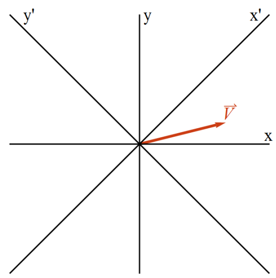

Figure 1

1. Write a matrix $\mathbf{C}$ that converts vectors from the xy basis to the $x'y'$ basis. In other words, write the components of $\mathbf{C}$ such that $\begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix} = \mathbf{C} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$. *Hint*: Try to see what the vector $<1,1>$ in the xy basis should become in the $x'y'$ basis and make sure your matrix $\mathbf{C}$ gets that one right.

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2. Write a matrix that converts vectors from the $x'y'$ basis to the xy basis. How is this matrix related to the matrix $\mathbf{C}$ that you just wrote?

3. Write a matrix $\mathbf{M}'$ that takes a vector in the $x'y'$ basis and stretches it by a factor of 2 in the x' direction. In other words $\mathbf{M}' \begin{pmatrix} V_{x'} \\ V_{y'} \end{pmatrix} = \begin{pmatrix} 2V_{x'} \\ V_{y'} \end{pmatrix}$.

4. Using your answers so far, write a matrix **M** that takes a vector in the xy basis, converts it to the $x'y'$ basis, stretches it by two in the x' direction, and then converts back to the xy basis.

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5. Describe in words what the matrix **M** does to a vector $\vec{V}$ in the xy basis. Your answer should make no reference to the $x'y'$ basis, but should just say what kind of stretching or reflection it performs. *Hint*: The answer does *not* involve a rotation.

6. Choose a vector on which the transformation you described should look very simple, and check that the matrix **M** you wrote down does what you would expect to that vector.