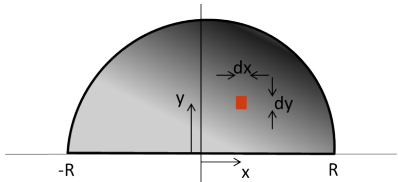


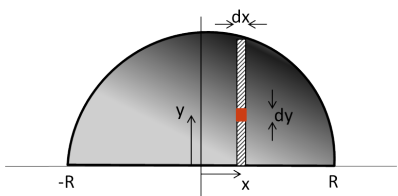
Discovery Exercise for Cartesian Double Integrals over a Non-Rectangular Region

A half-disk of radius R above the x -axis has a density given by $\sigma = kyx^2$ where k is a constant. Your goal is to find the total mass.

1. The drawing below shows a small rectangle at a specific x and y value with width dx and height dy . Write the area dA and the mass dm of this small region as a function of x , y , dx , and dy .



2. In the next drawing this small rectangle has been extended into a vertical strip. Write a function for the y -value at the top of this strip as a function of x .



3. To compute the mass of the entire vertical strip, integrate your dm from Part 1 as y goes from the bottom ($y = 0$) to the top (the function you found in Part 2). The result will be a function of x and dx .
4. Integrate your answer from Part 3 as x goes from $-R$ to R to compute the mass of the entire half-disk. Check that your answer has correct units.

See Check Yourself #29 at felderbooks.com/checkyourself