Discovery Exercise for Line Integrals

We begin with a fact from physics: if a force $\vec{F}$ acts upon an object that moves through a linear displacement $d\vec{s}$, the work done by the force on the object is given by the dot product $W = \vec{F} \cdot d\vec{s}$.

1. Calculate the work done by a force $\vec{F} = 2\hat{i} + 3\hat{j}$ on an object as it moves along a line from the point $(1, 1)$ to $(5, 25)$.

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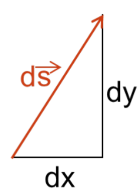
Part 1 does not require an integral, because a constant force is acting along a straight line. If the force or the path varies, you employ the usual strategy: break the journey into small parts, compute each part with a simple dot product like Part 1, and use an integral to add up all the contributions.

Consider, for instance, a bead moving along a wire in the shape of the curve $y = x^2$. A force $\vec{F} = (x + y)\hat{i} + (xy)\hat{j}$ acts on the bead as it moves from $(1, 1)$ to $(5, 25)$. Your job in this exercise is to calculate the work done by the force on the bead.

We begin with a small part of the journey—small enough to treat the force as a constant and the journey as a line.



2. The force is expressed as a function of x and y. Re-express the force—both its x and y components—as a function of x only. Remember what curve we are moving along!

3. The figure above represents a small displacement along the curve $d\vec{s} = dx\ \hat{i} + dy\ \hat{j}$. Express the relationship between dy and dx, based once again on the curve. (*Hint*: It's not $dy = dx^2$.)

4. Calculate the work done along this small displacement. Your answer should be expressed as a function of x and dx only.

5. Integrate your answer as x goes from 1 to 5.

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