Discovery Exercise for Parametrically Expressed Surfaces

A surface is described by the equations $x = u + v^2$, $y = u^2$, $z = v^2$ on the domain $u \geq 0$, $v \geq 0$. For instance, $u = 7$ and $v = 5$ correspond to the point $(32, 49, 25)$. Your job is to find *all* points that can result from plugging (u, v) points into these functions.

1. For $(u, v) = (0, 0)$, find the values of x, y, and z. Plot the corresponding point (x, y, z). (Note that your drawing is based only on x, y, and z, *not* on u and v.)

2. For $(u, v) = (1, 0)$, plot the appropriate point (x, y, z) on the same plot.
3. For $(u, v) = (1, 1)$, plot the appropriate point (x, y, z) on the same plot.

We can build up a lot of points this way, but let's try a faster method. We'll start by seeing what this graph looks like when $v = 0$. Because $z = v^2$, this means that $z = 0$; that is, we are seeing what this surface looks like on the xy plane.

4. For $v = 0$ find the functions $x(u)$ and $y(u)$.

5. Use those two functions to find a function $y(x)$ that has no u or v in it.

6. Graph the resulting function. The result is a picture of what this surface looks like on the xy plane.

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So instead of building up our graph one point at a time, we can go one curve at a time.

7. Plug $v = 1$ into the function for z to find what plane we are working on.

8. Plug $v = 1$ into the equations for x and y to find functions $x(u)$ and $y(u)$. Then use those functions to find a function $y(x)$.

9. Draw the curve that you found in Part 8 that shows what this surface looks like on the plane you found in Part 7.

10. Repeat Questions 7–9 for $v = 2$.

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11. Repeat Questions 7–9 for $v = 3$.

12. Draw and/or describe the surface you get when you include all real values of u and v.