Discovery Exercise for Slope Fields

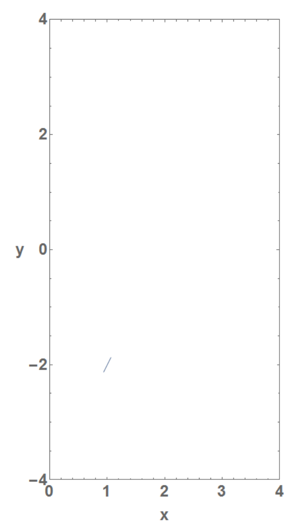
In this exercise, you will examine the differential equation:

$$\frac{dy}{dx} = |y| \tag{1}$$

As you know, there is not only one function that solves such a differential equation; there are many different solutions, corresponding to different initial conditions. So when we set out to view this differential equation graphically we will not end up with "the right curve" but with a set of curves, sometimes called a "family of solutions," all represented on one graph.

1. Equation 1 tells us "For any given point that the graph of a function goes through, here is its slope at that point." Based on that formula, what is the slope of a function that follows this differential equation as it passes through the point $(1, -2)$?

You should have found that the slope at that point would be 2. We can represent this visually by drawing a small line through the point $(1, -2)$ with a slope of 2. The length of this line segment is irrelevant; all we are trying to show is the slope.

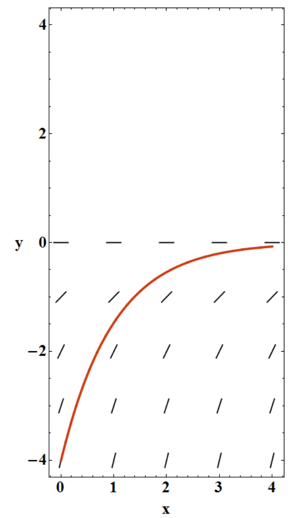


This drawing does *not* mean that the solution must go through the point $(1, -2)$. (There is in fact one solution that does go through that point, and infinitely many that don't.) The drawing also is *not* meant to suggest that the solution is linear, even for a short while. It simply says one solution rises with a slope of two through that point, and therefore momentarily looks something like the drawing.

2. On the graph on the attached page, draw lines similar to the one shown above for all integer points $0 \leq x \leq 4$ and $-4 \leq y \leq 0$ (a total of 25 points in the fourth quadrant).

3. Consider now the solution that begins at $(0, -4)$. The slope at that point is 4, so start moving to the right with a slope of four. As the curve rises, what happens to the slope? What happens to the function as a result? Draw the function that results.

You should have wound up with a drawing something like the following.



Keep in mind that this is not ultimately about drawing curves, but about predicting behavior. If y represents some meaningful quantity and x represents time we can say the following: if y starts at -4 it will increase rapidly at first and then gradually slow down, approaching but never reaching zero. We figured all this out visually, without ever *solving* the differential equation.

4. Consider now the solution that begins at $(0, 0)$. Draw this function as you drew the previous one. Then write a brief paragraph describing this solution, just as we described the $(0, -4)$ solution.

5. Draw in slope lines for all the points on $0 \leq x \leq 4$ and $1 \leq y \leq 4$ (first quadrant).

6. Consider now the solution that begins at $(0, 1)$. Draw this function as you drew the previous one. Then write a brief paragraph describing this solution, just as we described the $(0, -4)$ solution.