Discovery Exercise for Linear First Order Differential Equations

A linear first order differential equation can always be written in the form:

$$\frac{dy}{dx} + a_0(x)y = f(x) \tag{1}$$

1. It would seem that a more general form would be $a_1(x)y' + a_0(x)y = f(x)$. Why can we claim that Equation 1 is fully general?

2. Write the complementary homogeneous equation for Equation 1.

3. Solve your homogeneous equation by separation of variables. Your answer will have an integral with respect to x in it.

4. Confirm that your answer works.

There is a general method for solving the *inhomogeneous* Equation 1. As you might expect, the solution you just found for the complementary equation is a key part of that method.