Motivating Exercise for Methods of Solving ODEs: A Damped, Driven Oscillator

An object with mass m is attached to an ideal spring that exerts a force $F_{spring} = -kx$, but is also slowed down by a drag force $F_{drag} = -bv$.

1. Write the equation of motion for this object. That is, write the differential equation for $x(t)$ based on the given forces and Newton's law $F_{total} = ma$.

2. Now write the equation of motion assuming $m = 1$ kg, $b = 6$ N·s/m, and $k = 9$ N/m.

   See Check Yourself #62 at felderbooks.com/checkyourself

3. Show that $x = Ae^{-3t} + Bte^{-3t}$ is a valid solution to your differential equation for any values of the constants A and B. You will find such solutions yourself when you learn the techniques "guess and check" and "reduction of order"; right now we are just asking you to verify that it works.

4. What additional information would you need to determine the constants A and B? What kind of behavior does this solution describe, and why does this make sense for our scenario?

Now, in addition to the forces of the spring and drag, there is an external "driving force" F_e acting on the spring. In the following parts you will be writing differential equations that you do not yet know how to solve.

5. Write the equation for $x(t)$ for the driving force $F = t^5 e^{-3t}$. (Assume everything is in SI units. For example the 3 in the exponent is really 3 s^{-1}.) You will learn to solve equations like this one using the technique "variation of parameters."

6. Assume the driving force is a constant 3 N from $t = 0$ to $t = 10$ s and zero before and after that. Write the differential equation for $x(t)$. Right now you will need to use piecewise notation. You will learn to use the "Heaviside function" to represent such equations more conveniently, and "Laplace transforms" to solve them.

This exercise does not represent all the important techniques you will learn, but it should give you a sense that a simple real-world situation, with minor variations, can require many different approaches.