Discovery Exercise for Overview of Partial Differential Equations

We begin with an ordinary differential equation: that is, a differential equation with only one independent variable.

1. Consider the differential equation $dy/dx = y$. In words, "the function $y(x)$ is its own derivative."
   (a) Verify that $y = 2e^x$ is a valid solution to this differential equation.
   (b) Write another solution to this equation.
   (c) Write the *general* solution to this equation. It should have one arbitrary constant in it.
   (d) Find the only *specific* solution that meets the condition $y(0) = 7$.

Things become more complicated when differential equations involve functions of more than one variable, and therefore partial derivatives. For the following questions, suppose that z is a function of two independent variables x and y.

2. Consider the differential equation $\frac{\partial}{\partial x} z(x, y) = z(x, y)$. In words, "if you start with the function $z(x, y)$ and take its partial derivative with respect to x, you get the same function you started with." Note that $\partial z/\partial y$ is not specified by this differential equation, and may therefore be anything at all.
   (a) Which of the following functions are valid solutions to this differential equation? Check all that apply.
      i. $z = 5$
      ii. $z = e^x$
      iii. $z = e^y$
      iv. $z = e^x e^y$
      v. $z = ye^x$
      vi. $z = xe^y$
      vii. $z = e^x \sin y$
   (b) Write a *general* solution to the differential equation $\partial z/\partial x = z$. Your solution will have an *arbitrary function* in it.
   (c) Find the only *specific* solution that meets the condition $z(0, y) = \sin(y)$.

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3. Consider the differential equation $\frac{\partial}{\partial x}z(x,y) = \frac{\partial}{\partial y}z(x,y)$.

   (a) Express this differential equation in words.

   (b) Which of the following functions are valid solutions to this differential equation? Check all that apply.

      i. $z = 5$
      ii. $z = e^x$
      iii. $z = e^y$
      iv. $z = e^{x+y}$
      v. $z = \sin(x+y)$
      vi. $z = \sin(x-y)$
      vii. $z = \ln(x+y)$

   (c) Parts i, iv, v, and vii above are all specific examples of the general form $z = f(x+y)$. By plugging $z = f(x+y)$ into the original differential equation $\partial z/\partial x = \partial z/\partial y$, show that any function of this form provides a valid solution.

   (d) Find the only *specific* solution that meets the condition $z(0,y) = \cos y$. *Hint:* It's not in the list above.