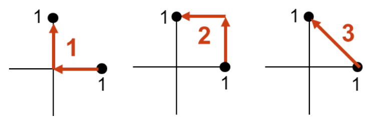
Discovery Exercise for Conservative Vector Fields

We begin by considering a very simple vector: $\vec{v}_1 = x\hat{i} + x\hat{j}$. You are going to find the line integral of this vector from $(1, 0)$ to $(0, 1)$ along three different paths.



1. Path 1 goes left to the origin and then straight up. Evaluate $\int_C \vec{v}_1 \cdot d\vec{s}$ along this path.

   See Check Yourself #56 at felderbooks.com/checkyourself

2. Path 2 goes up to $(1, 1)$ and then left. Evaluate $\int_C \vec{v}_1 \cdot d\vec{s}$ along this path.

3. Path 3 is a single straight line. Evaluate $\int_C \vec{v}_1 \cdot d\vec{s}$ along this path.

We now repeat the same exercise with a slightly more complicated vector: $\vec{v}_2 = 2xy\hat{i} + x^2\hat{j}$.

4. Evaluate $\int_C \vec{v}_2 \cdot d\vec{s}$ along path 1.

5. Evaluate $\int_C \vec{v}_2 \cdot d\vec{s}$ along path 2.

6. Evaluate $\int_C \vec{v}_2 \cdot d\vec{s}$ along path 3.

You should have found that $\int_C \vec{v}_1 \cdot d\vec{s}$ depends on the specific path you take, but $\int_C \vec{v}_2 \cdot d\vec{s}$ is "path-independent." Vectors with path-independent line integrals define a special case called "conservative" vectors, and you can evaluate their line integrals with the gradient theorem.

7. For the function $f(x, y) = x^2 y$ confirm that $\vec{\nabla} f = \vec{v}_2$.

8. Show that $f(0, 1) - f(1, 0)$ gives the same answer that you got with all three line integrals.