Motivating Exercise: An RLC Circuit

Figure 1.1 shows a circuit diagram with standard electrical symbols for a resistor, capacitor, and inductor.

The capacitor has charge Q_0 until the switch is closed, at which point the capacitor begins to discharge. The voltage drop across the resistor is IR, the voltage drop across the capacitor is Q/C, and the voltage drop across the inductor is $L(dI/dt)$. Because the voltage drop around the entire circuit must sum to zero, we can write $IR + Q/C + L(dI/dt) = 0$.

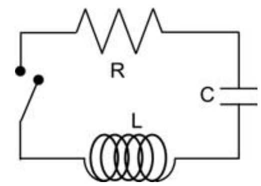

Figure 1.1: A simple RLC circuit

1. Remembering that $I = dQ/dt$, rewrite this equation as a second order differential equation for $Q(t)$.

2. Suppose the circuit elements are as follows: $R = 1010\ \Omega$, $C = 10^{-6}$ F, $L = 10^{-2}$ H.
   Plug in a guess of the form $Q(t) = e^{kt}$ and solve for k. You should find two possible values for k.

3. Using the values of k you just found, write the general solution to this differential equation for the given values of the constants. Remember that the general solution to a second order differential equation must have two arbitrary constants.

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The damping force from the resistor reduces the current rapidly; it decays exponentially until the capacitor is drained. Such a system is sometimes called "overdamped."

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4. Now consider a similar circuit with the following elements: $R = 1000\ \Omega$, $C = 2 \times 10^{-9}$ F, $L = 10^{-3}$ H. Rewrite the differential equation with these new constants, plug in the guess $Q(t) = e^{kt}$, and solve for k. You will find, in your solutions, the square root of a negative number. Just leave it there as part of your solutions, but simplify everything as much as possible.

5. Write the general solution to this differential equation, with the square root of a negative number as a part of your answer.

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A solution of this form, even if it works mathematically, does not seem to tell us anything about the charge on our capacitor. But in fact it tells us everything—once we know how to interpret it! When you learn about complex numbers, and especially about the complex exponential function, you will discover that this solution describes sinusoidal oscillation with decreasing amplitude. When you learn the physical properties of resistors, capacitors, and inductors, you will see why such behavior makes sense.