Discovery Exercise for Mapping Curves and Regions

A complex function can be described as a "mapping" from the complex plane to itself. That means that for every complex number that we put into the function we get a complex number out. To see what this means, consider the function $f(z) = z^2$.

1. If $z = 1 + i$ what is $f(z)$? *Hint:* it isn't zero.

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2. Draw a complex plane representing z. Mark on that plane the grid of points 0, 1, 2, i, $1 + i$, $2 + i$, $2i$, $1 + 2i$, and $2 + 2i$. Label the points "A," "B," and so on. Then draw another complex plane and mark the points $f(z)$ for each of the nine z values above, using the same labels.

As you can see, this technique represents one complex function with two different complex planes: one for the input z, and one for the output $f(z)$. Now draw two *more* complex planes, once again representing the input and output of $f(z) = z^2$. You will use these two for Parts 3–6.

3. On the z graph mark the line segment going from $z = 0$ to $z = 2$ (on the positive real axis). On the $f(z)$ graph mark the set of points f corresponding to all the z values on that line segment.

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4. Add a line segment from $z = 0$ to $z = 2i$ to the z graph and the corresponding set of points f on the f graph.

5. Add the curve $|z| = 2$ in the first quadrant to the z graph and the corresponding curve to the f graph.

6. The three curves you just drew on your z graph define a quarter of a disk. What shape does $f(z) = z^2$ map this quarter-disk to?