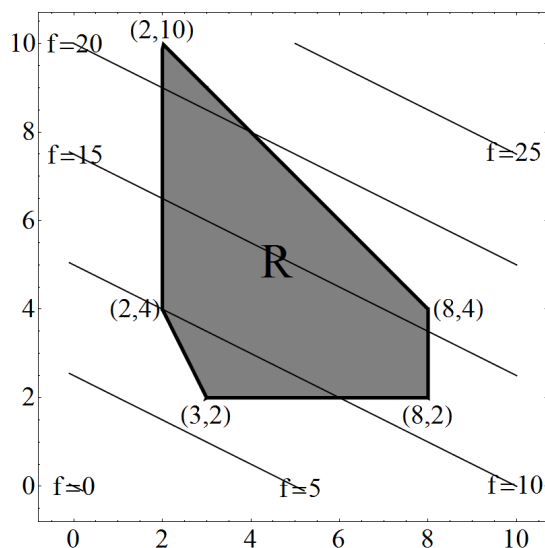


Discovery Exercise for Linear Programming and the Simplex Method

The picture shows a closed region R bounded by five lines. (The term “closed” indicates that the boundary lines are part of the region.) The picture also shows five contour lines of the function $f(x, y) = x + 2y$.



1. What point in region R has the largest possible value of $f(x, y)$?
2. What point in region R has the smallest possible value of $f(x, y)$?

See Check Yourself #47 at felderbooks.com/checkyourself

3. On the image of region R , draw in four contour lines of the function $g(x, y) = y - x$.
4. What point in region R has the largest value of g ? The smallest value?

You should have found that all four extrema occurred at the vertices of region R . When we know in advance that this is going to be true, we can search for minima and maxima by looking only at the vertices.

5. Let $h(x) = y + 2x$. The minimum of $h(x)$ within region R occurs at an infinite number of points. What points are those? Could we still find the maximum and minimum if we only looked at the vertices?
6. In many optimization problems the maximum or minimum is found inside the region, or along a border. Why did all the extrema in this exercise occur at the vertices? *Hint:* Your answer will involve a particular property of the functions $f(x)$, $g(x)$, and $h(x)$, and also a property of the region R .