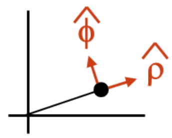
Discovery Exercise for Vectors in Curvilinear Coordinates

We begin our exercise at the point whose Cartesian coordinates are $(6, 3)$.



1. Give the polar coordinates ρ and ϕ for this point.

2. The vector $\hat{\rho}$ is a unit vector in the $+\rho$ direction: that is, a vector with magnitude 1 that points in the direction from the origin to the point. Give the Cartesian coordinates for $\hat{\rho}$ at this point.

   See Check Yourself #55 at felderbooks.com/checkyourself

3. The vector $\hat{\phi}$ is a unit vector in the $+\phi$ direction: that is, a vector of magnitude 1 that points tangent to a circle around the origin. At this particular point, $\hat{\phi} = \left(1/\sqrt{45}\right)\left(-3\hat{\imath} + 6\hat{\jmath}\right)$.

   (a) In order to qualify as $\hat{\phi}$ our vector must be a unit vector, and it must also be perpendicular to $\hat{\rho}$. (In other words, $\left|\hat{\phi}\right| = 1$ and $\hat{\rho} \cdot \hat{\phi} = 0$.) Show that our $\hat{\phi}$ meets both criteria.

   (b) The vector $\hat{\phi} = \left(1/\sqrt{45}\right)\left(3\hat{\imath} - 6\hat{\jmath}\right)$ also meets both criteria. How do we know that ours is correct and this one is wrong?

We move now to a more general point, the Cartesian (x, y).

4. Give the polar coordinates ρ and ϕ for this point as functions of x and y.

5. Give the Cartesian coordinates for $\hat{\rho}$ at this point.

6. Give the Cartesian coordinates for $\hat{\phi}$ at this point. *Hint:* Look at $\hat{\phi}$ at the point $(6, 3)$ in Part 3 and use that as a guide.

Finally, consider the vector field $\vec{f}(\rho, \phi) = 2\hat{\rho} - \hat{\phi}$. The following questions can be answered graphically—that is, without any difficult computation.

7. At the point $(1, 0)$, $\hat{\rho}$ points to the right and $\hat{\phi}$ points up. Compute $\vec{f}(\rho, \phi) = 2\hat{\rho} - \hat{\phi}$ in Cartesian coordinates at this point and draw it at this point.

8. At the point $(0, 1)$, $\hat{\rho}$ points up and $\hat{\phi}$ points to the left. Compute $\vec{f}(\rho, \phi) = 2\hat{\rho} - \hat{\phi}$ in Cartesian coordinates at this point and draw it at this point (on the same plot).

9. Compute and draw $\vec{f}$ at three other points.